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# Superconformal Sigma Models in Higher Than Two Dimensions

Ergin Sezgin[†]

*Center for Theoretical Physics, Texas A&M University*
*College Station, TX 77843–4242, USA*

and

Yoshiaki Tanii

*Physics Department, Saitama University*
*Urawa, Saitama 338, Japan*

## ABSTRACT

Rigidly superconformal sigma models in higher than two dimensions are constructed. These models rely on the existence of conformal Killing spinors on the $p+1$ dimensional worldvolume ($p \leq 5$), and homothetic conformal Killing vectors in the $d$–dimensional target space. In the bosonic case, substituting into the action a particular form of the target space metric admitting such Killing vectors, we obtain an action with manifest worldvolume conformal symmetry, which describes the coupling of $d-1$ scalars to a conformally flat metric on the worldvolume. We also construct gauged sigma models with worldvolume conformal supersymmetry. The models considered here are generalizations of the singleton actions on $S^p \times S^1$, constructed sometime ago by Nicolai and these authors.

# 1. Introduction

The importance of two dimensional superconformal field theories in the context of string theory is well known. In particular, superconformal sigma models in two dimensions play an important role in the first quantized description of string theory. In the manifestly world-sheet supersymmetric formulation, the string action has, in fact, local superconformal supersymmetry, which becomes rigid upon gauge fixing. As is well known, the resulting superconformal group is infinite dimensional, due to special aspects of two dimensional manifolds.

It is natural to search for superconformal sigma models in higher than two dimensions. Although the superconformal group becomes finite dimensional, it is nonetheless interesting to study these models in their own right. In particular, they may have an application in the description of the theory of super-extended objects, known as super p-branes. In fact, Nicolai and these authors [1], and independently Blencowe and Duff [2], sometime ago conjectured that super $p$–branes in an $AdS_{p+2} \times S^{N-1}$ background, where AdS refers to anti de Sitter space, are described by superconformal field theories on $S^p \times S^1$. They are sometimes referred to supersingleton field theories and can be viewed as $N$-extended superconformal sigma models in $p + 1$ dimensions, with a *flat* target space whose dimension is given by the number of real scalar fields. The possible values of $p$, $N$, number of scalars and global superconformal symmetries are tabulated below (the dimension of spacetime in which the $p$–brane propagates is $d = p + 1 +$ number of scalars).

| $p$ | $N$ | Number of Scalars | Superconformal Group |
|-----|--------|-------------------|----------------------|
| 1 | $1, 2, 4, 8$ | $1, 2, 4, 8$ | $OSp(N|2)$ |
| 2 | $1, 2, 4, 8$ | $1, 2, 4, 8$ | $OSp(N|4)$ |
| 3 | $1, 2, 4$ | $2, 4, 6$ | $SU(2, 2|N)$ |
| 4 | $2$ | $4$ | $F(4)$ |
| 5 | $2, 4$ | $4, 5$ | $OSp(6, 2|N)$ |

In a separate development, Gibbons and Townsend [3] showed that a number of supersymmetric $p$–brane solutions to $d = 10$ and $d = 11$ supergravity theories interpolate between Minkowski spacetime and $AdS_{p+2} \times S^{N-1}$ type compactified spacetime (the $p = 1$ case has been described recently in ref. [4]). These authors have argued that their results imply that the effective action for small fluctuations of the super $p$–brane is a supersingleton field theory. This raises the question as to whether supersingleton theories can also exist with target spaces other than the Euclidean space, and in some way related to the $p$–brane solutions mentioned above.

Motivated by the results of ref. [3,4], we are thus led to search for a more general class of $N$–extended superconformal sigma models in $p+1$ dimensions. We take the worldvolume to be any $p+1$ dimensional space with superconformal isometries, and we determine the conditions on the target space as required by the worldvolume superconformal symmetry [†]. While we do not provide a complete classification of superconformal sigma models in higher than two dimensions, we do, however, derive a general set of conditions for their existence. Since the superconformal groups exists in dimensions up to six (coinciding with the maximum worldvolume dimension allowed for super $p$–branes), we need to consider sigma models with worldvolume of $p+1$ dimensions with $p = 1, ..., 5$. Our results can be summarized briefly as follows.

The existence of rigidly superconformal sigma models in higher than two dimensions relies on the existence of conformal Killing vectors and spinors on the $p+1$ dimensional worldvolume ($p \leq 5$), and *homothetic conformal Killing vectors* in the $d$–dimensional target space. The latter are conformal Killing vectors which leave the metric $g$ invariant up to a *constant* conformal scale, i.e. $\mathcal{L}_\xi g = \lambda g$, where $\lambda$ is a constant [5] [††]. In the bosonic case, substituting into the action a particular form of the target space metric admitting such conformal Killing vectors, we obtain an action with manifest worldvolume conformal symmetry, which describes the coupling of $d-1$ scalars to a conformally flat metric in $p+1$ dimensions. In the supersymmetric case, we shall concentrate on the $p = 2, N = 1$ and $p = 5, N = 2$ cases, but the general structure will become clear for all the cases. [†††]. As for the worldvolume geometry, in the case of $p = 2, N = 1$, we shall consider a general worldvolume which has a conformal Killing spinor ($S^2 \times S^1$ is a particular case), while for the $p = 5, N = 2$ case, we shall take the worldvolume to be $S^5 \times S^1$.

Considering the superconformal sigma models in which the $d$ dimensional target space admits isometries which form a group $G$, one can gauge $G$ or any subgroup of it. In this paper, we also construct a gauged sigma model of this kind. Such sigma models may be of considerable interest in the context of duality transformations, which are essentially obtainable by integrating over suitable set of gauge fields.

In Sec. 2, we shall discuss the conformally invariant bosonic sigma models in arbitrary dimensions, and show the emergence of a manifestly conformally invariant model in one less

[†]   We use the terminology of *worldvolume* and *target space* in referring to the *domain* and *range* manifolds, respectively, of the sigma models.

[††]   In particular, group manifolds do not admit homothetic conformal Killing vectors. Therefore, we can not write down a singleton action as a conformally invariant sigma model with group manifold as a target space. On the other hand, "nonabelian singletons" have been considered in ref. [6]. It is not clear to us how the singleton Lagrangian of ref. [6] can be interpreted as a conformally invariant sigma model on a group manifold.

[†††]  For a study of the relation between the local super Weyl invariance and target space rigid superconformal invariance of super Weyl invariant version of super $p$–branes, see ref. [7], where it is shown that the two symmetries are incompatible.

target space dimension. In Sec. 3, we shall describe the $N = 1$ superconformal sigma model in a general 2+1 dimensional worldvolume and a general target space. We will derive the conditions imposed on the target space metric and other functions occurring in the action, by the requirement of worldvolume superconformal invariance. In Sec. 4, we will assume that the target space admits isometries and gauge these isometries. In Sec. 5, we will construct the $N = 2$ superconformal sigma model on $S^5 \times S^1$. Again we will derive the conditions imposed on the target space metric by the worldvolume superconformal invariance. In Sec. 6, we recapitulate our results and furthermore discuss an alternative approach to obtaining rigidly superconformal sigma models, namely from conformal supergravities, giving examples from $d = 6, N = 2$ conformal supergravity.

## 2. Conformal Sigma Models in Arbitrary Dimensions

We shall consider field theories consisting of real scalar fields $\phi^a$, $(a = 1, ..., d)$ on a worldvolume with metric $h_{ij}$, $i = 1, ..., p + 1$ that admits conformal Killing vectors. A conformal Killing vector $\xi^i$ satisfies

$$\nabla_i \xi_j + \nabla_j \xi_i = 4\Omega h_{ij} \ . \tag{2.1}$$

From (2.1), and recalling the Bianchi identity $\nabla^i(R_{ij} - \frac{1}{2}h_{ij}R) = 0$, we learn that

$$p\nabla^i \partial_i \Omega + R\Omega + \frac{1}{4}\xi^i \partial_i R = 0 \ . \tag{2.2}$$

Now let us consider a bosonic theory in the general world volume which admits a conformal Killing vector:

$$\mathcal{L} = -\frac{1}{2}\sqrt{-h}\left[h^{ij}\partial_i\phi^a\partial_j\phi^b G_{ab}(\phi) + RV(\phi) + U(\phi)\right] \ , \tag{2.3}$$

where $G_{ab}$, $V$ and $U$ are functions of the scalar fields $\phi^a$. The bosonic sector of the known supersingleton theory corresponds to a special case of this Lagrangian. (Eq. (2.7) below, together with the condition that $R = p(p-1)$, as appropriate for $S^p \times S^1$). The conformal transformation of the scalar fields is defined by using $\xi$ and $\Omega$ satisfying eq. (2.1) as

$$\delta_C \phi^a = \xi^i \partial_i \phi^a + \Omega v^a(\phi) \ , \tag{2.4}$$

where $v^a$ are functions of the scalar fields. These transformations satisfy the closed conformal algebra $[\delta(\xi_1), \delta(\xi_2)] = \delta(\xi_3)$ where $\xi_3^i = \xi_2^j \partial_j \xi_1^i - \xi_1^j \partial_j \xi_2^i$.

Conformal transformation of the Lagrangian (2.3) up to total derivative terms becomes

$$\begin{aligned}
\delta\mathcal{L} = -\frac{1}{2}\sqrt{-h}\Big[&\Omega h^{ij}\partial_i\phi^a\partial_j\phi^b\left(D_a v_b + D_b v_a - 2(p-1)G_{ab}\right) \\
&+ \Omega\left(v^a\partial_a U - 2(p+1)U\right) + \Omega R\left(v^a\partial_a V - 2(p-1)V\right) \\
&+ 2h^{ij}\partial_i\Omega\partial_j\phi^a(v_a - 2p\partial_a V)\Big] \ ,
\end{aligned} \tag{2.5}$$

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where we have used (2.1) and (2.2). Therefore, the condition for conformal invariance of the Lagrangian is [†]

$$D_a v_b + D_b v_a = 2(p-1)G_{ab} \ , \tag{2.6a}$$

$$v_a = 2p\partial_a V \ , \tag{2.6b}$$

$$v^a \partial_a U = 2(p+1)U \ , \tag{2.6c}$$

$$v^a \partial_a V = 2(p-1)V \ . \tag{2.6d}$$

Eq. (2.6a) means that $v^a$ is a *homothetic Killing vector*. As mentioned earlier, group manifolds do not admit homothetic conformal Killing vectors. However, a solution of the above equations with a flat target space metric exists and is given by

$$G_{ab} = \delta_{ab} \ , \qquad v^a = (p-1)\phi^a \ , \qquad V = \frac{p-1}{4p}\delta_{ab}\phi^a\phi^b \ , \tag{2.7}$$

and $U$ an arbitrary homogeneous function of $\phi^a$ of order $2(p+1)/(p-1)$. Note that the metric for the worldvolume need not be $S^p \times S^1$, but it can be any space admitting ordinary conformal Killing vectors satisfying (2.1).

To solve eq. (2.6) in general, we split the target space coordinates as $\phi^a = (\phi^0, \phi^\alpha)$ ($\alpha = 1, \cdots, d-1$). We then choose a coordinate system in which $G_{0a} = 2p\partial_a V$ [††]. In this coordinate system, the condition eqs. (2.6a) and (2.6b) reduce to $(p-1)G_{ab} = 2pD_a\partial_b V = \partial_0 G_{ab}/2$. This is easily integrated to solve for $G_{ab}$. Using this result, the solution of (2.6a) and (2.6b) is found to be

$$\begin{aligned} V &= e^{2(p-1)\phi^0}\bar{V}(\phi^\alpha) \ , \\ G_{00} &= 4p(p-1)e^{2(p-1)\phi^0}\bar{V}(\phi^\alpha) \ , \\ G_{0\alpha} &= 2pe^{2(p-1)\phi^0}\partial_\alpha\bar{V}(\phi^\alpha) \ , \\ G_{\alpha\beta} &= e^{2(p-1)\phi^0}\bar{G}_{\alpha\beta}(\phi^\alpha) \ , \end{aligned} \tag{2.8}$$

where $\bar{V}(\phi^\alpha)$ and $\bar{G}_{\alpha\beta}(\phi^\alpha)$ are *arbitrary* functions of $\phi^\alpha$. Eq. (2.8) also satisfies eq. (2.6d). Using eq. (2.8), eq. (2.6c) becomes $\partial_0 U = 2(p+1)U$ , which is solved to yield

$$U = e^{2(p+1)\phi^0}\bar{U}(\phi^\alpha) \ , \tag{2.9}$$

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[†]   A similar set of conditions appeared in a study of the Weyl invariance of sigma models coupled to dynamical metric in general dimensions [8].

[††]   This choice is possible because $G_{00}$ and $G_{0\alpha}$ are essentially the lapse and shift functions that arise in the canonical formulation of general relativity, and it is known that they can be set equal to arbitrary fixed functions by a coordinate transformation, at least locally.

where $\bar{U}(\phi^\alpha)$ is also an *arbitrary* function of $\phi^\alpha$. Substituting the solution (2.8) and (2.9), the Lagrangian (2.3) becomes

$$\mathcal{L} = -\frac{1}{2}\sqrt{-h}\Big[h^{ij}\partial_i\phi^\alpha\partial_j\phi^\beta e^{2(p-1)\phi^0}\bar{G}_{\alpha\beta}(\phi^\alpha) + 4ph^{ij}\partial_i\phi^\alpha\partial_j\phi^0 e^{2(p-1)\phi^0}\partial_\alpha\bar{V}(\phi^\alpha)$$
$$+ 4p(p-1)h^{ij}\partial_i\phi^0\partial_j\phi^0 e^{2(p-1)\phi^0}\bar{V}(\phi^\alpha) + e^{2(p-1)\phi^0}R\bar{V}(\phi^\alpha) + e^{2(p+1)\phi^0}\bar{U}(\phi^\alpha)\Big] .$$
$$(2.10)$$

If we define

$$\tilde{h}_{ij} = e^{-4\phi^0}h_{ij} ,\qquad(2.11)$$

the action (2.10) can be written as

$$\mathcal{L} = -\frac{1}{2}\sqrt{-\tilde{h}}\Big[\tilde{h}^{ij}\partial_i\phi^\alpha\partial_j\phi^\beta\bar{G}_{\alpha\beta}(\phi^\alpha) + \tilde{R}\bar{V}(\phi^\alpha) + \bar{U}(\phi^\alpha)\Big] ,\qquad(2.12)$$

where, we recall that $\bar{G}_{\alpha\beta}$, $\bar{V}$ and $\bar{U}$ are arbitrary functions of $\phi^\alpha$. This is the action of the matter scalar fields $\phi^\alpha$ coupled to the metric $\tilde{h}_{ij}$. The dependence on $\phi^0$ has been completely absorbed into the conformal mode of $\tilde{h}_{ij}$. The conformal transformation of the fields can be written as

$$\delta_C\phi^\alpha = \xi^i\partial_i\phi^\alpha ,$$
$$\delta_C\tilde{h}_{ij} = \tilde{\nabla}_i\tilde{\xi}_j + \tilde{\nabla}_j\tilde{\xi}_i ,\qquad(2.13)$$

where $\tilde{\xi}_i = \tilde{h}_{ij}\xi^j$. These conformal transformations have the same form as the general coordinate transformation of scalar fields and a metric. Therefore, the action (2.12) is manifestly conformally invariant.

Finally, we note that the general solution leading to (2.10) contains the flat space solution given by (2.7). Substituting the latter solution into the Lagrangian (2.3) we obtain

$$\mathcal{L}_{\text{flat}} = -\frac{1}{2}\sqrt{-h}\Big[h^{ij}\partial_i\phi^a\partial_j\phi^a + \frac{p-1}{4p}R(\phi^a)^2 + U(\phi^a)\Big] ,\qquad(2.14)$$

where, we recall that $U$ is an arbitrary homogeneous function of $\phi^a$ of order $2(p+1)/(p-1)$. This Lagrangian can be transformed into the form (2.10) by a change of the target space coordinates $\phi^a \to (\phi^0, \phi^\alpha)$:

$$\phi^a = e^{(p-1)\phi^0}\hat{\phi}^a(\phi^\alpha) \qquad (a = 0, 1, \cdots, d-1),\qquad(2.15)$$

where $\hat{\phi}^a$ satisfy $\hat{\phi}^a\hat{\phi}^a = 1$ and are parametrized by the coordinates of $\mathrm{S}^{d-1}$: $\phi^\alpha$ ($\alpha = 1, \cdots, d-1$). In terms of the new coordinates $\phi^0, \phi^\alpha$ the Lagrangian is given by

$$\mathcal{L}_{\text{flat}} = -\frac{1}{2}\sqrt{-h}\Big[h^{ij}\partial_i\phi^\alpha\partial_j\phi^\beta e^{2(p-1)\phi^0}\bar{G}_{\alpha\beta} + (p-1)^2 h^{ij}\partial_i\phi^0\partial_j\phi^0 e^{2(p-1)\phi^0}$$
$$+ \frac{p-1}{4p}e^{2(p-1)\phi^0}R + e^{2(p+1)\phi^0}U(\hat{\phi}^a(\phi^\alpha))\Big] ,\qquad(2.16)$$

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where $\bar{G}_{\alpha\beta}$ is the round metric of $S^{d-1}$. We see that this is a special case of eq. (2.10), in which

$$\bar{V}(\phi^\alpha) = \frac{p-1}{4p} \ , \qquad \bar{U}(\phi^\alpha) = U(\hat{\phi}^a(\phi^\alpha)) \ . \tag{2.17}$$

We now turn to the supersymmetrization of the Lagrangian (2.3).

## 3. General Superconformal Sigma Model in 2+1 Dimensions

In ref. [1], supersingleton field theories on $S^2 \times S^1$ with flat $N$ dimensional target space were constructed for $N \leq 8$. Here, we shall generalize that model by taking the worldvolume to be a general 2+1 dimensional space which admits conformal Killing spinors, and target space to be arbitrary. For simplicity, we shall restrict our attention to the scalar multiplet of $N = 1$ superconformal symmetry. The scalar supermultiplets consist of real scalar fields $\phi^a$ $(a = 1, \cdots, M)$ and Majorana spinor fields [†] $\lambda^A$ $(A = 1, \cdots, M)$.

To proceed with the construction of the transformation rules and the action, it is essential to have conformal Killing spinors [9]. A conformal Killing spinor $\eta_-$ in $p + 1$ dimensional space satisfies

$$\nabla_i \eta_- - \frac{1}{2}\gamma_i \eta_+ = 0 \ . \tag{3.1}$$

From this equation we obtain

$$p\gamma^i \nabla_i \eta_+ + \frac{1}{2}R\eta_- = 0 \ , \tag{3.2}$$

where we have used $\nabla^i \nabla_i \eta_- = \frac{1}{2}\gamma^i \nabla_i \eta_+$, which can be derived from eq. (3.1) [††].

Now let consider the following generalization of the supersingleton Lagrangian of ref. [1]:

$$\mathcal{L} = -\frac{1}{2}\sqrt{-h}\Big[h^{ij}\partial_i\phi^a\partial_j\phi^b G_{ab} + RV + U - i\bar{\lambda}^A\gamma^i(\nabla_i\lambda^B + \partial_i\phi^a\omega_a{}^B{}_C\lambda^C)\delta_{AB}$$
$$- iV_{AB}\bar{\lambda}^A\lambda^B + \frac{1}{4}\Omega_{ABCD}\bar{\lambda}^A\gamma^i\lambda^B\bar{\lambda}^C\gamma_i\lambda^D\Big] \ , \tag{3.3}$$

where $G_{ab}$, $V$, $U$, $\omega_a{}^A{}_B$, $V_{AB}$ and $\Omega_{ABCD}$ are to be determined by superconformal invariance. The first three terms in eq. (3.3) constitute the bosonic Lagrangian considered in the previous section, while the known supersingleton action corresponds to a special case of the Lagrangian (3.3) (Eq. (3.11) below, together with the condition that $R = 2$, as appropriate

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[†]   We use two-component spinors, which are equivalent to four-component spinors with a certain type of chirality condition used in ref. [1].

[††]   The solutions of the conformal Killing spinor equation (3.1) exist on $S^p \times S^1$, and they were used in the formulation of supersingleton field theories in ref. [1]. It is interesting to note that by considering the integrability conditions of eq. (3.1), one finds that a conformal Killing spinor must satisfy the equation $C_{ijkl}\gamma^{kl}\eta_- = 0$, where $C_{ijkl}$ is the Weyl tensor [9].

for $S^2 \times S^1$ ). The conformal transformations of the fields are defined by using $\xi$ and $\Omega$ satisfying eq. (2.1) as

$$\delta_C \phi^a = \xi^i \partial_i \phi^a + \Omega v^a ,$$

$$\delta_C \lambda^A = \xi^i \nabla_i \lambda^A + \frac{1}{4} \nabla_i \xi_j \gamma^{ij} \lambda^A + 2\Omega \lambda^A + \Omega Q^A{}_B \lambda^B , \tag{3.4}$$

where $v^a$ and $Q_A{}^B$ are functions of the scalar fields. For *arbitrary* functions $v^a$ and $Q_A{}^B$ these transformations satisfy the closed conformal algebra $[\delta(\xi_1), \delta(\xi_2)] = \delta(\xi_3)$ where $\xi_3^i = \xi_2^j \partial_j \xi_1^i - \xi_1^j \partial_j \xi_2^i$. Once we establish the superconformal invariance of the action, its invariance under the bosonic conformal transformations will be guarantied, since the anticommutator of the former yields the latter (see eq. (3.7) below). Thus, we now turn our attention to the superconformal symmetries of the Lagrangian (3.3).

The supertransformations of the fields are defined by using $\eta_\pm$ satisfying (3.1) as

$$\delta_Q \phi^a = -i \bar{\eta}_- \lambda^A e_A{}^a,$$

$$\delta_Q \lambda^A = \gamma^i \partial_i \phi^a \eta_- e_a{}^A - \delta_Q \phi^a \omega_a{}^A{}_B \lambda^B - m^A \eta_- + \frac{1}{2} v^A \eta_+ , \tag{3.5}$$

where we have introduced the new functions $e_A{}^a$, $e_a{}^A$, $m^A$ and $v^A$. We first require that the commutator of two supertransformations (3.5) closes up to the equations of motion. This requires

$$e_a{}^A e_A{}^b = \delta_a{}^b , \tag{3.6a}$$

$$D_a e_b{}^A - D_b e_a{}^A = 0 , \tag{3.6b}$$

$$e_a{}^A = D_a v^A, \tag{3.6c}$$

$$\Omega_{ABCD} = \frac{1}{3} e_A{}^a e_B{}^b R_{abCD} , \tag{3.6d}$$

$$V_{AB} = D_A m_B , \tag{3.6e}$$

$$Q^A{}_B = -v^a \omega_a{}^A{}_B , \tag{3.6f}$$

where $D_a$ and $R_{abCD}$ are the covariant derivative and the Riemann tensor respectively defined by the spin connection $\omega_a{}^A{}_B$ [†]. The commutator algebra is

$$[\delta_Q(\eta_1), \delta_Q(\eta_2)] = \delta_C(\xi) , \quad \xi^i = -2i \bar{\eta}_{2-} \gamma^i \eta_{1-} , \tag{3.7}$$

where the conformal transformation $\delta_C$ is as defined in eq. (3.4) [††]. The invariance of the Lagrangian (3.3) under the supertransformation (3.5) further requires

$$G_{ab} = e_a{}^A e_b{}^B \eta_{AB} , \tag{3.8a}$$

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[†] It may be useful to note that eq. (3.6c) has a rather strong integrability condition which reads: $v^a R_{abCD} = 0$.

[††] Note that the existence of conformal Killing spinors implies the existence of conformal Killing vectors as follows: When $\eta_{1-}, \eta_{2-}$ satisfy the conformal Killing spinor equation (3.1), $\xi^i$ in eq. (3.7) satisfies the conformal Killing equation (2.1) with $\Omega$ given by $\Omega = \frac{1}{2} i (\bar{\eta}_{2+} \eta_{1-} - \bar{\eta}_{2-} \eta_{1+})$.

$$3m_A = V_{AB}v^B \ , \tag{3.8b}$$

$$e_A{}^a\partial_a U = 2V_{AB}m^B \ , \tag{3.8c}$$

$$e_A{}^a\partial_a V = \frac{1}{4}v_A \ . \tag{3.8d}$$

We can obtain the general solution of eqs. (3.6) and (3.8). First, eqs. (3.6a), (3.6b), (3.6d), (3.8a) are trivially solved. Next, the other conditions are solved by

$$v_a = 4\partial_a V \ , \tag{3.9a}$$

$$G_{ab} = 4D_a\partial_b V \ , \tag{3.9b}$$

$$m_a = \partial_a m \ , \tag{3.9c}$$

$$V_{ab} = D_a\partial_b m \ , \tag{3.9d}$$

$$v^a\partial_a m = 4m \ , \tag{3.9e}$$

$$U = m_a m_b G^{ab} \ . \tag{3.9f}$$

Note that, the conditions (2.6) for the bosonic conformal invariance are automatically satisfied by the above solution. Eqs. (3.9a,b) agree with (2.6a,b), while to see that (2.6c,d) are satisfied, note from (3.9e,c,b) that $\mathcal{L}_v m_a = 4m_a$ and $\mathcal{L}_v G^{ab} = -2G^{ab}$. Hence, from (3.9f) one finds the result $\mathcal{L}_v U = 6U$, which agrees with (2.6c). To see that (2.6d) is satisfied, we multiply (3.9b) with $v^b$ from which it follows that $v^a = \frac{1}{2}\partial_a(v^b v_b)$. Comparing with (3.9a) we learn that $v^a v_a = 8V$, which agrees with (2.6d).

We still have to find functions $V$, $G_{ab}$ and $m$ which satisfy $G_{ab} = 4D_a\partial_b V$ and $v^a\partial_a m = 4m$. They can be obtained as in the bosonic case. The general solution up to target space coordinate transformations is

$$\begin{aligned}
m &= e^{4\phi^0}\bar{m}(\phi^\alpha) \ , \\
V &= e^{2\phi^0}\bar{V}(\phi^\alpha) \ , \\
G_{00} &= 8e^{2\phi^0}\bar{V}(\phi^\alpha) \ , \\
G_{0\alpha} &= 4e^{2\phi^0}\partial_\alpha\bar{V}(\phi^\alpha) \ , \\
G_{\alpha\beta} &= e^{2\phi^0}\bar{G}_{\alpha\beta}(\phi^\alpha) \ ,
\end{aligned} \tag{3.10}$$

where $\bar{V}(\phi^\alpha)$, $\bar{m}(\phi^\alpha)$ and $\bar{G}_{\alpha\beta}(\phi^\alpha)$ are arbitrary functions of $\phi^\alpha$. Substituting this solution into the Lagrangian (3.3), we have not been able to cast the resulting Lagrangian in a manifestly superconformally invariant form, as we did in the bosonic case. However, we do expect that be possible, and to give rise to supergravity coupled to $M-1$ scalar multiplets, where the only dynamical degrees of freedom in the supergravity multiplet are the conformal mode of the metric and the superconformal mode of the Rarita-Schwinger field.

Finally, we note that a particular solution of (3.9) with flat target space metric is

$$e_a{}^A = \delta_a^A \ , \qquad V = \frac{1}{8}\delta_{ab}\phi^a\phi^b \ , \qquad m = \frac{1}{4}C_{abcd}\phi^a\phi^b\phi^c\phi^d \ , \tag{3.11}$$

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where $C_{abcd}$ is an arbitrary constant coefficient which is totally symmetric in its indices. Note that the metric for the worldvolume need not be $S^2 \times S^1$, but it can be any space admitting ordinary conformal Killing vectors satisfying (2.1).

## 4. Gauging of Isometries of the Superconformal Sigma Model in 2+1 Dimensions

Let G be the isometry group or its subgroup of the metric $G_{ab}$ in the Lagrangian (3.3). There exist Killing vectors $K_r^a$ ($r = 1, \cdots$, dimG) satisfying the Killing equation

$$D_a K_{rb} + D_b K_{ra} = 0 \tag{4.1}$$

and commutation relations of the Lie algebra of G

$$[K_r, K_s] = if_{rs}{}^t K_t, \quad K_r = K_r^a \partial_a. \tag{4.2}$$

By applying $D_c$ to eq. (4.1) and antisymmetrizing the indices $c$ and $a$, we obtain a useful identity

$$K_r^d R_{dabc} = D_a D_b K_{rc}. \tag{4.3}$$

We define rigid isometry transformations corresponding to the Killing vectors $K_r^a$ by

$$\begin{aligned}
\delta\phi^a &= \epsilon^r K_r^a, \\
\delta\lambda^A &= \epsilon^r (D_B K_r^A - K_r^a \omega_a{}^A{}_B)\lambda^B,
\end{aligned} \tag{4.4}$$

where $\epsilon^r$ are infinitesimal constant parameters. The transformation of the spinor fields looks simpler if we use $\lambda^a \equiv \lambda^A e_A{}^a$: $\delta\lambda^a = \epsilon^r \partial_b K_r^a \lambda^b$. The commutator algebra of these transformations closes. The Lagrangian (3.3) with eqs. (3.6) and (3.8) is invariant under the transformations (4.4) if the coupling functions satisfy

$$K_r^a \partial_a V = 0, \quad K_r^a \partial_a m = 0 \tag{4.5}$$

in addition to eq. (4.1). To prove the invariance of the action we need an identity

$$\begin{aligned}
K_r^a D_a R_{ABCD} &+ D_A K_r^E R_{EBCD} + D_B K_r^E R_{AECD} \\
&+ D_C K_r^E R_{ABED} + D_D K_r^E R_{ABCE} = 0,
\end{aligned} \tag{4.6}$$

which can be shown by using the Bianchi identity and eq. (4.3). The first condition in eq. (4.5) is equivalent to the condition that the vector fields $v = v^a \partial_a$ and $K_r = K_r^a \partial_a$ commute each other. For a flat space solution (3.11), we can take the group G to be SO($M$). The Killing vectors are $K^a = \lambda^a{}_b \phi^b$, where $\lambda_{ab} = -\lambda_{ba}$ are constant parameters. The condition (4.5) requires that the coefficient $C_{abcd}$ in eq. (3.11) is an invariant tensor of SO($M$).

We would like to make the theory invariant under local isometry transformations, i.e., the transformations (4.4) with parameters $\epsilon_r(x)$ of arbitrary functions of $x^i$. We introduce

gauge supermultiplets consisting of vector fields $A_i^r$ and Majorana spinor fields $\chi^r$. The gauge transformations $\delta_g$ of the fields are given by eq. (4.4) and

$$\delta_g A_i^r = \partial_i \epsilon^r + i f^r{}_{st} A_i^s \epsilon^t,$$
$$\delta_g \chi^r = i f^r{}_{st} \chi^s \epsilon^t. \tag{4.7}$$

We define the covariant derivatives

$$D_i \phi^a = \partial_i \phi^a - A_i^r K_r^a,$$
$$D_i \lambda^A = \nabla_i \lambda^A + D_i \phi^a \omega_a{}^A{}_B \lambda^B - A_i^r (D_B K_r^A - K_r^a \omega_a{}^A{}_B) \lambda^B \tag{4.8}$$
$$= \nabla_i \lambda^A + \partial_i \phi^a \omega_a{}^A{}_B \lambda^B - A_i^r D_B K_r^A \lambda^B,$$

which transform under the gauge transformations (4.4) and (4.7) as

$$\delta_g (D_i \phi^a) = \epsilon^r \partial_b K_r^a D_i \phi^b,$$
$$\delta_g (D_i \lambda^A) = \epsilon^r (D_B K_r^A - K_r^a \omega_a{}^A{}_B) D_i \lambda^B. \tag{4.9}$$

The supertransformations of the scalar and the gauge multiplets are given by

$$\delta_Q \phi^a = -i \bar{\eta}_- \lambda^A e_A{}^a,$$
$$\delta_Q \lambda^A = \gamma^i D_i \phi^a \eta_- e_a{}^A - \delta_Q \phi^a \omega_a{}^A{}_B \lambda^B - m^A \eta_- + \frac{1}{2} v^A \eta_+,$$
$$\delta_Q A_i^r = i \bar{\eta}_- \gamma_i \chi^r, \tag{4.10}$$
$$\delta_Q \chi^r = \frac{1}{2} F_{ij}^r \gamma^{ij} \eta_-.$$

The commutator algebra of the supertransformations (4.10) closes and is given by

$$[\delta_Q(\eta_1), \delta_Q(\eta_2)] = \delta_C(\xi) + \delta_g(\epsilon),$$
$$\xi^i = -2i \bar{\eta}_{2-} \gamma^i \eta_{1-}, \tag{4.11}$$
$$\epsilon^r = -\xi^i A_i^r,$$

where the conformal transformations of the gauge multiplets are

$$\delta_C A_i^r = \xi^j \nabla_j A_i^r + \nabla_i \xi^j A_j^r,$$
$$\delta_C \chi^r = \xi^i \nabla_i \chi^r + \frac{1}{4} \nabla_i \xi_j \gamma^{ij} \chi^r - 3\Omega \chi^r. \tag{4.12}$$

Notice that the conformal weight of $\chi$ is different from that of $\lambda$ in eq. (3.4). It should also be noted that the algebra closes off-shell on the gauge multiplets. We do not need to use equations of motion of these fields to obtain the algebra (4.11). This can be understood from the fact that a gauge field and a Majorana spinor field has the same off-shell degrees of freedom in three dimensions.

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The gauged Lagrangian is taken to be

$$
\begin{aligned}
\mathcal{L}_{\text{gauged}} = -\frac{1}{2}\sqrt{-h}\Big[ & h^{ij}D_i\phi^a D_j\phi^b G_{ab} + RV + U - i\bar{\lambda}^A\gamma^i D_i\lambda_A \\
& - 2i\bar{\lambda}^A\chi^r K_{rA} - iV_{AB}\bar{\lambda}^A\lambda^B + \frac{1}{4}\Omega_{ABCD}\bar{\lambda}^A\gamma^i\lambda^B\bar{\lambda}^C\gamma_i\lambda^D\Big].
\end{aligned}
\tag{4.13}
$$

The kinetic terms of the gauge multiplets $-\frac{1}{4}\sqrt{-h}F^r_{\mu\nu}F^{r\mu\nu}$ and $\sqrt{-h}\bar{\chi}^r i\gamma^i D_i\chi^r$ have not been included, since they are not invariant under the conformal transformations (4.12). The Lagrangian (4.13) is invariant under the conformal, the gauge and the supersymmetry transformations when the conditions (3.6), (3.8), (4.1) and (4.5) are satisfied.

## 5. Superconformal Sigma Model in 5+1 Dimensions

In ref. [1], we considered the $N = 2$ supersingleton field theory on $S^5 \times S^1$ with four dimensional flat target space. Here, we shall consider a generalization of the model by taking the target space to be an arbitrary manifold, and find the conditions imposed on it by the requirement of the worldvolume superconformal invariance.

The $N = 2$ supermultiplet consist of real scalar fields $\phi^a$ ($a = 1, \cdots, 4M$) and symplectic Majorana-Weyl spinor fields $\lambda^A_+$ ($A = 1, \cdots, 2M$):

$$
\lambda^A_+ = \Omega^{AB}C\bar{\lambda}^T_{+B}, \quad \gamma_7\lambda^A_+ = \lambda^A_+,
\tag{5.1}
$$

where $\Omega^{AB} = -\Omega^{BA}$ is a constant matrix. We use $\Omega^{AB}$ and $\Omega_{AB}$ defined by $\Omega^{AB}\Omega_{BC} = \delta^A{}_C$ to raise and lower indices. The Lagrangian is

$$
\begin{aligned}
\mathcal{L} = -\frac{1}{2}\sqrt{-h}\Big[ & h^{ij}\partial_i\phi^a\partial_j\phi^b G_{ab} + U \\
& + i\bar{\lambda}^A_+\gamma^i(\nabla_i\lambda_{+A} + \partial_i\phi^a\omega_{aAB}\lambda^B_+) + \frac{1}{4}\Omega_{ABCD}\bar{\lambda}^A_+\gamma^i\lambda^B_+\bar{\lambda}^C_+\gamma_i\lambda^D_+\Big].
\end{aligned}
\tag{5.2}
$$

Notice that the Yukawa coupling $V_{AB}\bar{\lambda}^A_+\lambda^B_+$ is not possible due to the chirality of the spinor fields. The coefficient functions have symmetry properties $\omega_{aAB} = \omega_{aBA}$ and $\Omega_{ABCD} = \Omega_{BACD} = \Omega_{CDAB}$. The conformal transformations of the fields are defined by using $\xi$ and $\Omega$ satisfying eq. (2.1) (with $p = 5$) as

$$
\begin{aligned}
\delta_C\phi^a &= \xi^i\partial_i\phi^a + \Omega v^a, \\
\delta_C\lambda^A &= \xi^i\nabla_i\lambda^A + \frac{1}{4}\nabla_i\xi_j\gamma^{ij}\lambda^A + 5\Omega\lambda^A + \Omega Q^A{}_B\lambda^B,
\end{aligned}
\tag{5.3}
$$

while the supertransformations of the fields are

$$
\begin{aligned}
\delta_Q\phi^a &= i\bar{\eta}^I_-\lambda^A_+ e_{IA}{}^a, \\
\delta_Q\lambda^A_+ &= \gamma^i\partial_i\phi^a\eta_{-I}e_a{}^{IA} - \delta_Q\phi^a\omega_a{}^A{}_B\lambda^B_+ + 2v^{IA}\eta_{+I}.
\end{aligned}
\tag{5.4}
$$

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The conformal Killing spinors $\eta_\pm^I$ ($I = 1, 2$) satisfying eq. (3.1) are symplectic Majorana-Weyl

$$\eta_\pm^I = \Omega^{IJ} C \bar{\eta}_{\pm J}^T \ , \quad \gamma_7 \eta_\pm^I = \pm \eta_\pm^I, \tag{5.5}$$

where $\Omega^{IJ} = -\Omega^{JI}$ is a constant matrix and $\Omega^{IJ} \Omega_{JK} = \delta^I{}_K$.

The closure of the commutator algebra of eq. (5.4) and the invariance of the Lagrangian under eq. (5.4) require that

$$
\begin{aligned}
e_a{}^{IA} e_{IA}{}^b = \delta_a{}^b \ , \quad G_{ab} = e_a{}^{IA} e_b{}^{JB} \Omega_{IJ} \Omega_{AB} \ , \\
\partial_a e_b{}^{IA} + \omega_a{}^A{}_B e_b{}^{IB} - (a \leftrightarrow b) = 0 \ , \\
\partial_a v^{IA} + \omega_a{}^A{}_B v^{IB} = e_a{}^{IA} \ , \\
e_{IA}{}^a \partial_a U = 8 v_{IA} \ , \\
e_{IA}{}^a e_{JB}{}^b R_{abCD} = 6 \Omega_{IJ} \Omega_{ABCD} \ , \\
Q^A{}_B = -v^a \omega_a{}^A{}_B \ , \\
v^a = v^{IA} e_{IA}{}^a \ .
\end{aligned}
\tag{5.6}
$$

We also need the fact that $\Omega_{ABCD}$ is totally symmetric in the indices, which can be shown by the Bianchi identity and the sixth condition of eq. (5.6). The commutator algebra is

$$
\begin{aligned}
[\delta_Q(\eta_1), \delta_Q(\eta_2)] = \delta_C(\xi) + \delta_{SU(2)}(\Lambda), \\
\xi^i = i \bar{\eta}_{2-}^I \gamma^i \eta_{1-I}, \\
\Lambda^{\alpha\beta} = -2i \left( \bar{\eta}_{2-}^I (\Gamma^{\alpha\beta})_I{}^J \eta_{1+J} - \bar{\eta}_{1-}^I (\Gamma^{\alpha\beta})_I{}^J \eta_{2+J} \right) ,
\end{aligned}
\tag{5.7}
$$

where $(\Gamma_\alpha)_I{}^J$ ($\alpha = 1, 2, 3$) are the SO(3) $\gamma$-matrices. The SU(2) automorphism transformations are defined by

$$
\begin{aligned}
\delta_{\text{SU}(2)} \phi^a = \frac{1}{4} \Lambda^{\alpha\beta} v^{IA} (\Gamma_{\alpha\beta})_I{}^J e_{JA}{}^a, \\
\delta_{\text{SU}(2)} \lambda_+^A = -\delta_{\text{SU}(2)} \phi^a \omega_a{}^A{}_B \lambda_+^B.
\end{aligned}
\tag{5.8}
$$

Note that once the conditions (5.6) are satisfied, the invariance of the Lagrangian under the bosonic conformal transformations (5.3) is guarantied, because the superconformal algebra (5.7) has been verified. In fact, while the conditions (2.6) are sufficient, the necessary conditions that follow from invariance under (5.3) will look somewhat different than those given in (2.6), because $R = $ constant for $S^p \times S^1$. We need not write down those conditions here, because they are simply consequences of the conditions given in eq. (5.6).

Finally, we note that for a flat target space metric the general solution of eq. (5.6) is [1]

$$e_a{}^{IA} = \delta_a{}^{IA}, \quad \omega_a{}^A{}_B = 0, \quad \Omega_{ABCD} = 0, \quad v^{IA} = \phi^{IA}, \quad U = 4 \phi^{IA} \phi_{IA}. \tag{5.9}$$

Interaction terms in the potential $U$ is not possible.

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## 6. Conclusions

We have constructed superconformal sigma models which generalize the known supersingleton field theories. We have found that the superconformal sigma model in 2+1 dimensions is given by

$$\mathcal{L} = -\frac{1}{2}\sqrt{-h}\Big[h^{ij}\partial_i\phi^a\partial_j\phi^b G_{ab} + RV + \partial_a m\,\partial_b m\,G^{ab} - i\bar{\lambda}^A\gamma^i(\nabla_i\lambda^B + \partial_i\phi^a\omega_a{}^B{}_C\lambda^C)\delta_{AB}$$
$$ - i(D_a\partial_b m)\,\bar{\lambda}^a\lambda^b + \frac{1}{12}R_{abcd}\bar{\lambda}^a\gamma^i\lambda^b\bar{\lambda}^c\gamma_i\lambda^d\Big]\ ,$$

$$(6.1)$$

where $m$, $V$ and the metric $G_{ab}$ are given in (3.10), $\lambda^a = e^a{}_A\lambda^A$, the covariant derivative $D_a$ and the curvature $R_{abCD} = R_{abcd}e^c{}_C e^d{}_D$ are defined with respect to the spin connection $\omega_{aA}{}^B$. The Lagrangian has the following superconformal symmetry:

$$\delta_Q\phi^a = -i\bar{\eta}_-\lambda^A e_A{}^a,$$
$$\delta_Q\lambda^A = \gamma^i\partial_i\phi^a\eta_- e_a{}^A - \delta_Q\phi^a\omega_a{}^A{}_B\lambda^B - e^{Aa}\partial_a m\eta_- + 2e^{aA}\partial_a V\eta_+\ ,$$

$$(6.2)$$

where the parameters $\eta_\pm$ satisfy the conformal Killing spinor equation (3.1). Splitting the scalar fields as $\phi^a = (\phi^0, \phi^\alpha)$ $(\alpha = 1, \cdots, d-1)$, from (3.10) we observe that the Lagrangian and transformation rules depend on three arbitrary functions of $\phi^\alpha$, namely $\bar{m}$, $\bar{V}$ and $\bar{G}_{\alpha\beta}$. The corresponding result in ref. [1] is a special case of the result above, in which the worldvolume is taken to be $S^2 \times S^1$, the $d$-dimensional space is flat and $m$, $V$ are specific functions defined in (3.11).

We have gauged the isometries of the target space manifold characterized by the Killing vectors $K_r^a$, and obtained the Lagrangian given in (4.13), which is invariant under the conformal, the gauge, and the supersymmetry transformations when the conditions (3.6), (3.8), (4.1) and (4.5) are satisfied.

In the case of a 5+1 dimensional worldvolume, we have restricted our attention to $S^5 \times S^1$, but considered an arbitrary target space. In that case, we have found the Lagrangian (5.2), invariant under (5.3) and (5.4), when the conditions (5.6) are satisfied.

No doubt results similar to those presented here will also hold for $p + 1$ dimensional worldvolumes with all values of $p \leq 5$. It should be noted, however that we have found essentially no restrictions on the dimensions of the possible target spaces (apart from the fact that in the case of $p = 5$, the target space dimension is a multiple of four). Thus, the critical dimensions of the super $p$–branes is somewhat mysterious in the context of superconformal sigma models presented here. It is intriguing to speculate that quantum consistency of our models may lead to certain critical dimensions. In fact, it would be interesting to work out the quantum behaviour of our models in its own right. We hope to return to this point in the future.

Finally, let us note that there exists an alternative way to obtain particular kinds of superconformal sigma models, which may have been left out of the class considered here.

Namely, one could start with a conformal supergravity theory coupled to scalar fields, and fix a superconformal gauge in such a way that a *rigid* superconformal symmetry is maintained and that the only dynamical degrees of freedom are those of the scalar multiplet, possibly together with a Liouville type supermultiplet of fields corresponding to the conformal modes of the Weyl supermultiplet.

To illustrate this last point, let us consider the conformal supergravity theory in $d = 6$, which is the highest dimension where a superconformal group exists. The $d = 6, N = 2$ conformal supergravity and its coupling to various multiplets has been studied in [10]. The most natural multiplet to consider here is the hypermultiplet, consisting of the scalar fields $\phi^{IA}$, $I = 1, 2; A = 1, ..., 2M+2$ and the superpartners $\lambda_A$. To obtain a rigid superconformal sigma model, we choose a superconformal gauge by fixing the gravitational field such that it admits a conformal Killing spinor (see eq. (3.1)), and set all the other gauge fields of the Weyl supermultiplet equal to zero. In particular, setting the gravitino field $\psi_i^I$ equal to zero implies eq. (3.1), since $\delta\psi_i^I = \nabla_i \eta_-^I - \frac{1}{2}\gamma_i \eta_+^I$, where $\eta_-^I$ is the ordinary supersymmetry parameter and $\eta_+^I$ is the special supersymmetry parameter. In this way, we find

$$\mathcal{L} = -\frac{1}{2}\sqrt{-h}\left[h^{ij}\partial_i\phi^{IA}\partial_j\phi_{IA} + \frac{1}{5}R\phi^{IA}\phi_{IA} + i\bar{\lambda}_+^A\gamma^i\nabla_i\lambda_{+A}\right] . \tag{6.3}$$

The superconformal symmetry of this Lagrangian is characterized by the transformations given in (5.3) and (5.4), with $e_a{}^{IA} = \delta_a^{IA}$ and $v^{IA} = \phi^{IA}$ ($I = 1, 2$).

Applying the above procedure to the gauged version of the $d = 6, N = 2$ conformal supergravity, we find that the field equations of the resulting rigid superconformal sigma model are unacceptable, because they force the scalar fields to vanish.

It may be worth mentioning that there are two other $d = 6, N = 2$ superconformal matter multiplets that contain scalar fields. One of them is the nonlinear multiplet [10], and it contains three scalars of zero Weyl weight, parametrizing an $SU(2)$ group manifold, and a real constrained vector field. The other one is the linear multiplet [10] and it contains three scalar fields which have Weyl weight four, and a fourth rank totally antisymmetric tensor field which is equivalent to a scalar on-shell. Using the action formula provided in [10] for the coupling of the linear multiplet to $d = 6, N = 2$ conformal supergravity, we can obtain a rigid superconformal sigma model for this multiplet by fixing a superconformal gauge as described above. The fermionic terms in the resulting Lagrangian are rather involved, but the bosonic sector is simple and is given by:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\sqrt{-h}\left[\phi^{-1}\partial_i\phi^{IJ}\partial^i\phi_{IJ} + \phi R + \frac{1}{240}\phi^{-1}H_{i_1\cdots i_5}H^{i_5\cdots i_5}\right]\\&+ \frac{1}{48}\epsilon^{i_1\cdots i_6}B_{i_1\cdots i_4}\left(\partial_{i_5}\phi^{IJ}\right)\phi_J{}^K\left(\partial_{i_6}\phi_{KI}\right) ,\end{aligned} \tag{6.4}$$

where $\phi_{IJ} = \phi_{JI}$, $\phi = \left(\phi^{IJ}\phi_{IJ}\right)^{1/2}$, and $B$ is the four-form with field strength $H = dB$.

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